

BROWN-HET-961
to appear in MNRAS

Statistics of Peculiar Velocities from Cosmic Strings

R. Moessner¹

Department of Physics
Brown University
Providence RI. 02912.
USA.

Abstract

We calculate the probability distribution of a single component of peculiar velocities due to cosmic strings, smoothed over regions with a radius of several h^{-1} Mpc. The probability distribution is shown to be Gaussian to good accuracy, in agreement with the distribution of peculiar velocities deduced from the 1.9 Jy IRAS redshift survey. Using the normalization of parameters of the cosmic string model from CMB measurements, we show that the rms values for peculiar velocities inferred from IRAS are consistent with the cosmic string model provided that long strings have some small-scale structure.

key words: cosmic strings ; large-scale structure of Universe ; cosmology:theory

¹e-mail address moessner@het.brown.edu ; address from October: Max Planck Institut für Astrophysik, Karl-Schwarzschild-Str. 1, 85740 Garching bei München, Germany

1 Introduction

Cosmic strings might be responsible for the formation of large-scale structure which is observed in the Universe today. They are topological defects which form in a phase transition in the early universe. Topological defect models of structure formation are an alternative to inflationary models. In inflationary models one assumes the existence of a scalar field which drives a phase of very rapid expansion in the early universe, and quantum fluctuations produced during this phase turn into classical density perturbations which grow by gravitational instability into present-day structures. In topological defect models the defects, which are present at all times including today (and could possibly be seen directly some day), act as seeds around which structures form. It is important to study theories from both classes of models in order to see which predictions can distinguish between different models, and which predictions are too generic, arising in models based on widely different assumptions.

Among the topological defect models, cosmic strings were the first to be investigated in more detail (Brandenberger 1991), recently textures have also been subjected to closer scrutiny (Turok 1991). The cosmic string model looks promising so far, since normalizations of the parameters occurring in the model obtained from observation agree with each other (Bennet, Bouchet & Stebbins 1992, Perivolaropoulos 1993a). These observations include galaxy redshift surveys and measurements of the temperature anisotropies in the cosmic microwave background radiation (CMB). Moreover, cosmic strings naturally produce filaments and planar structures in the matter distribution (Silk & Vilenkin 1984, Vachaspati 1986, Stebbins et al. 1987), in encouraging agreement with recent galaxy redshift surveys. Another nice feature of the cosmic string model is that it works well in the context of hot dark matter (Brandenberger 1987), for which there are candidates known to exist (the μ and τ neutrinos), although they may be massless. In contrast, none of the candidates for cold dark matter required in inflationary models has been detected.

Topological defect models have not been as closely investigated as inflationary

models, and more work is necessary to quantify the predictions of the cosmic string model. It is especially important to find features accessible to observations in which cosmic strings differ from the class of inflationary models, and statistics sensitive to these features. Recently, the probability distribution of peculiar velocities has been put forward as a statistic which might be capable of discriminating between cosmic strings and inflation (Kofman et al. 1994). Here we show that according to the calculations in a simple string toy model, this statistic cannot be used to differentiate between cosmic strings and inflation.

The IRAS 1.9 Jy redshift survey was analyzed (Strauss et al. 1990 & 1992, Yahil 1991) to obtain a uniform galaxy-density map. The peculiar velocities were reconstructed from it via a self-consistent iterative scheme assuming linear biasing between the density fluctuations of galaxies and mass (Nusser et al. 1991). Using these results, Kofman et al. (1994) determined the probability distribution of a single component of peculiar velocities of regions smoothed over several h^{-1} Mpc, with the result that it is consistent with the underlying distribution $p(v_x)$ being a normal distribution. Their result for the Gaussianity of the velocity distribution is still tentative because of the limited volume sampled, and because velocities were not measured directly but deduced from a redshift survey.

Inflationary models predict a normal distribution for $p(v_x)$, whereas some deviation from gaussianity is expected in the cosmic string model since individual strings impart coherent velocity perturbations over extended regions, as we will see in the following section. The question is, however, just how large this deviation from gaussianity is and whether it is big enough to be detected by current observations. For a certain region receives velocity perturbations from many strings, and by the central limit theorem many nongaussian perturbations can add up to a gaussian signal. Scherrer (1992) has shown that for seed models the velocity field can be very nearly Gaussian even if the density field is nongaussian. In a model where randomly distributed point masses, all of the same mass, accrete matter gravitationally in a

universe dominated by hot dark matter, he found that a very low seed density of less than 10^{-2}Mpc^{-3} is required in order for $p(v_x)$ to be nongaussian.

The main aim of this paper is to calculate $p(v_x)$ within the cosmic string model in order to quantify its departure from gaussianity, and to find out if the IRAS results are also in agreement with this stringy probability distribution $p(v_x)$. Previously, 3 dimensional rms velocities have been obtained analytically within the cosmic string model (Vachaspati 1992, Perivolaropoulos & Vachaspati 1994), and numerical simulations have been performed (Vachaspati 1992, Hara, Mähönen & Miyoshi 1993) to obtain the probability distribution of peculiar velocities, and it was found that the distribution is quite gaussian. Here we present the first analytical calculation of $p(v_x)$, within a model for the string network which has been previously employed to make predictions about temperature anisotropies in the CMB (Perivolaropoulos 1993a & 1993b, Moessner, Perivolaropoulos & Brandenberger 1994) and the magnitude of peculiar velocities (Vachaspati 1992, Perivolaropoulos & Vachaspati 1994). The dependence of the deviation from gaussianity on one of the parameters of the model, namely the number ν of strings per Hubble volume in the strings' scaling solution, is shown explicitly.

Our conclusion is that on scales of several h^{-1} Mpc, $p(v_x)$ from strings deviates only slightly from a normal distribution. On the smallest scales and for the smallest number of strings per Hubble volume, the largest deviation from gaussianity is expected. Performing a χ^2 -test of the stringy $p(v_x)$ for the smallest scale of $6h^{-1}$ Mpc given in Kofman et al. (1994), with imaginary data binned in the same way as in that paper, but drawn from an underlying gaussian distribution (worst case), yields the result that the stringy probability distribution is in agreement with these data. This agreement holds for all values of ν , the number of long strings per Hubble volume in the strings' scaling solution, including $\nu = 1$. This shows that the hope expressed in Kofman et al. (1994) that the scenario for the formation of large scale structure, where widely separated strings accrete matter in wakes behind them, can be ruled

out using the statistic $p(v_x)$ is not realized.

In the next section we will describe the cosmic string model and the mechanism for the production of velocity perturbations. In the third section we will describe the analytic model for the string network and calculate the moment generating function for a single component of the peculiar velocities averaged over several h^{-1} Mpc. Then we will obtain the probability distribution $p(v_x)$ from this moment generating function, and compare it with observations.

2 Cosmic Strings and Structure Formation

Cosmic strings are linear topological defects formed at a phase transition in the very early universe (Vilenkin 1985). Those originating in the symmetry breaking of a Grand Unified Theory possess an enormous mass per unit length μ ($G\mu \approx 10^{-6}$, where G is Newton's constant), and they can be responsible for the structures observed today. Strings can have no ends, so they are formed as either infinitely long strings or closed loops. After formation the network of strings quickly evolves towards a scaling solution where the energy density in long strings remains a constant fraction of the total background energy density. This is achieved by intercommutations and self-intersections of the strings leading to the production of small loops, which then decay by emitting gravitational radiation. In this way, some of the energy input into the string network coming from the stretching of the strings due to the expansion of the universe is transferred to the background. Long strings are straight over distances of the order of the horizon, so that the scaling solution can be pictured as having a fixed number ν of long strings per Hubble volume at any given time.

Initially loops were thought to make the dominant contribution to structure formation. At distances much larger than their radius they act as point masses and accrete surrounding matter (Turok & Brandenberger 1986, Stebbins 1986, Sato 1986). Improved cosmic string evolution simulations (Bennet & Bouchet 1988, Albrecht &

Turok 1989, Allen & Shellard 1990) showed that more of the energy density is in long strings, and they are therefore more important, accreting matter in the form of wakes behind them as they move through space (Brandenberger, Perivolaropoulos & Stebbins 1990, Perivolaropoulos, Brandenberger & Stebbins 1990). Spacetime around a long straight cosmic string can be pictured as locally flat, but with a deficit angle of $8\pi G\mu$ (Vilenkin 1985). Therefore a string moving relativistically with velocity v_s imparts velocity perturbations to surrounding matter towards the plane swept out by the string. If small-scale structure is present on the string, there is in addition a Newtonian force towards the string. The magnitude of this velocity perturbation is given by (Vachaspati & Vilenkin 1991, Vollick 1992)

$$u = 4\pi G\mu\gamma_s v_s f, \quad f = 1 + \frac{1 - T/\mu}{2(\gamma_s v_s)^2} \quad (1)$$

In the absence of small-scale structure on the string, the tension T of the string is equal to its mass per unit length μ , and $f = 1$. If small-scale structure is present, μ denotes the mass per unit length obtained after averaging over the small scale structure, $T \neq \mu$, and $f > 1$. We consider the perturbations caused by strings after t_{eq} , the time of equal matter and radiation, in a universe filled with hot or cold dark matter. By the present time, the initial velocity perturbation imparted to the dark matter at time t_i has grown to (Brandenberger 1987, Stebbins 1987, Hara & Miyoshi 1990)

$$u_i \approx 0.4u\sqrt{z(t_i)} \quad (2)$$

Due to compensation (Traschen, Turok & Brandenberger 1986, Veeraraghavan & Stebbins 1992), the deficit angle of strings which are straight over a horizon distance, extends out only to a distance of one Hubble radius H^{-1} from the string, so that matter which is farther away does not receive any velocity perturbations. The velocity perturbation given in eq.(1) is independent of distance from the string (up to the Hubble radius), so that cosmic strings impart coherent perturbations over regions of the size of half a Hubble volume.

3 Moment Generating Function

The moment generating function (mgf) $M_X(t)$ of a random variable X is defined by

$$M_X(t) = \langle \exp^{tX} \rangle \quad (3)$$

where the brackets denote the ensemble average, and it contains complete information about X . In the following, $X = v_x$ denotes the random variable for the component of the peculiar velocities smoothed over a region \mathcal{V} of comoving radius R in a fixed direction \hat{e}_x . We will calculate the mgf of X in order to obtain the moments (and cumulants) and probability distribution $p(v_x)$ of X from it. The moments of X , $\mu_j = \langle X^j \rangle$ are given by

$$\mu_j = \left(\frac{d^j}{dt^j} \right)_{t=0} M_X(t) \quad (4)$$

and the cumulants c_j are defined by

$$c_j = \left(\frac{d^j}{dt^j} \right)_{t=0} \ln(M_X(t)) \quad (5)$$

The probability distribution $p(v_x)$ can be expanded in an asymptotic series called Edgeworth series (Scherrer & Bertschinger 1991, Stuart & Ord 1987) in terms of the quantities

$$\lambda_j = c_j / c_2^{j/2} \quad (6)$$

and Hermite polynomials $H_n(x)$ defined by

$$H_n(x) = (-1)^n e^{x^2/2} \frac{d^n}{dx^n} e^{-x^2/2} \quad (7)$$

For distributions with vanishing odd moments as in our case, the expansion is

$$p(\delta) = \frac{e^{-\delta^2/2}}{\sqrt{2\pi}} \left[1 + \frac{\lambda_4}{24} H_4(\delta) + \frac{\lambda_6}{720} H_6(\delta) + \frac{\lambda_8 + 35\lambda_4^2}{40320} H_8(\delta) + \dots \right] \quad (8)$$

where $\delta = v_x / \sigma$, and σ is the standard deviation of X . Since it is an asymptotic expansion, the remainder is of the order of the last term included (Erdelyi 1956). The mgf has an important property which makes it useful for calculations. For

independent random variables X and Y , the mgf of the sum is the product of the individual moment generating functions

$$M_{X+Y}(t) = M_X(t)M_Y(t) \quad (9)$$

The following calculations are carried out within an analytical model for the string network which has previously been used to obtain the temperature anisotropies in the CMB and the magnitude of peculiar velocities from strings (see references given in the introduction). According to the scaling solution for cosmic strings, there is a fixed number of long strings present per Hubble volume at any given time. After about one expansion time of the universe (Hubble time), $t \rightarrow 2t$, these strings will typically have self-intersected or intercommuted, so that the resulting strings are uncorrelated with the ones at the previous Hubble time. We will assume that during one expansion time ν long strings move across the Hubble volume. Each string is assumed to be straight over one horizon distance, and the effect of all strings is taken to be the superposition of the effects of the individual strings. The fact that small-scale structure varies along strings is neglected, so that we might somewhat underestimate the degree of non-Gaussianness of the velocity distribution. We also assume that the strings' positions, velocities and orientations at each Hubble time are random and uncorrelated, although this is not strictly true, since - to mention one reason - the string network has the form of a self-avoiding random walk. According to the two previous assumptions, the random variable for the total peculiar velocity, X , is the sum of independent random variables for the velocity perturbations from the individual strings. By eq.(9) we can therefore reduce the calculation of the the mgf for X to that of the mgf for the contribution of only one string, and take the products afterwards. In fact we know that there are ν strings per Hubble volume on average. The products in eq.(9) simplify if we take a Poisson distribution for the number of strings per Hubble volume instead of assuming the presence of exactly ν of them (Scherrer & Bertschinger 1993). We can picture this as having a reservoir of n strings, each with a probability ν/n of being present in a particular Hubble volume,

in the limit that $n \rightarrow \infty$. So if $M_{Y_i}(t)$ denotes the mgf for a single component of peculiar velocities of the region \mathcal{V} due to one string present at time t_i in the region's Hubble volume, then

$$\begin{aligned} M_{X_i}(t) &= \lim_{n \rightarrow \infty} \left[\frac{\nu_i}{n} \langle \exp^{tY_i} \rangle + \left(1 - \frac{\nu_i}{n}\right)^n \right] \\ &= \exp[\nu_i(M_{Y_i}(t) - 1)] \end{aligned} \quad (10)$$

where X_i is the random variable for the contribution of all strings at Hubble time t_i to the velocity perturbation of \mathcal{V} , and ν_i denotes the average number of strings having an effect on \mathcal{V} at time t_i , i.e. those strings which are within a distance of one Hubble radius of \mathcal{V} at time t_i (see eq.(18)).

Since $X = \sum_{i=1}^N X_i$, where N is the number of expansion times since t_{eq}

$$N = \log_2 \frac{t_0}{t_{eq}} \quad (11)$$

and the X_i are assumed to be independent,

$$\begin{aligned} M_X(t) &= \prod_{i=1}^N M_{X_i}(t) \\ &= \exp \left[\sum_{i=1}^N \nu_i (M_{Y_i}(t) - 1) \right] \end{aligned} \quad (12)$$

We will now calculate the mgf for Y_i , the random variable for the component in the fixed direction \hat{e}_x of the peculiar velocities of a region \mathcal{V} of comoving radius R (the smoothing radius) due to one string affecting the region at time t_i . We have to perform the ensemble average over positions, orientations \hat{e}_s and directions of velocity \hat{v}_s of the string.

For a long straight string only transverse velocities are observable, and we assumed \hat{e}_s and \hat{v}_s to be random unit vectors. Therefore the unit normal $\hat{e} = \hat{e}_s \times \hat{v}_s$ of the plane swept out by the string, i.e. the direction in which matter receives velocity perturbations, is itself a random unit vector. Consequently, the projection $s = \hat{e} \cdot \hat{e}_x$ is uniformly distributed over the interval $[-1, 1]$, and the magnitude of the velocity

perturbation from one string has to be multiplied by s to get the component in direction \hat{e}_x .

During one expansion time a string sweeps out a plane towards which matter within a distance of one Hubble radius receives velocity perturbations, which have grown to u_i (see eq.(2)) by today. The possible values y_i for the random variable Y_i , the projection of the peculiar velocity of \mathcal{V} in direction \hat{e}_x due to one string at time t_i , are a function of the perpendicular distance r of the centre of \mathcal{V} to this plane:

$$\begin{aligned} y_i(r) &= su_i r / R_i \quad \text{for } 0 < r < R_i \\ y_i(r) &= su_i \quad \text{for } R_i < r < H_i^{-1} - R_i \\ y_i(r) &= su_i \frac{H_i^{-1} - (r - R_i)}{2R_i} \quad \text{for } H_i^{-1} - R_i < r < H_i^{-1} + R_i \end{aligned} \quad (13)$$

R_i is the physical size of the comoving radius R at time t_i . Strings within a distance of $H^{-1}(t_i) \equiv H_i^{-1}$ of the region \mathcal{V} can affect it. We distribute the centres C of these planes randomly within a sphere of radius $r_{max}^i = H_i^{-1}$ around the centre of \mathcal{V} . So the probability $p(c)$ for C to be a distance c from the centre of \mathcal{V} is

$$p(c) = \frac{3c^2}{(r_{max}^i)^3} \quad (14)$$

Since the normal of this plane has random direction, r can be smaller or equal to c , with probability

$$p(r; c) \approx 2 \frac{r}{c^2} \quad (15)$$

Integrating over all c gives the probability for the plane to be a distance r from the centre of \mathcal{V} as

$$p(r) = \int_r^{r_{max}^i} dc p(r; c) p(c) = \frac{6r}{(r_{max}^i)^3} (r_{max}^i - r) \quad (16)$$

The ensemble average thus becomes an integral over r and s

$$\begin{aligned} M_{Y_i}(t) &= \langle \exp^{tY_i} \rangle \\ &= \int_0^{r_{max}^i} dr p(r) \int_{-1}^1 ds \frac{1}{2} \exp(ty_i(r)) \end{aligned} \quad (17)$$

These integrals can be done, and then eq.(12) can be used to obtain the mgf for X which includes the effect of all strings, with the number of strings affecting \mathcal{V} at time t_i being given by

$$\nu_i = \nu(r_{max}^i)^3 / (H_i^{-1})^3 \quad (18)$$

There is one problem with the above. The formulas for $y_i(r)$ in equations (13) are only true if the projection of \mathcal{V} onto the plane swept out by the string along its normal falls completely into that plane, and is not (partly) outside of it. But the latter can happen for large c for some orientations of the plane, since one side of the plane, in the direction of the string's motion, has a length $l_i = H_i^{-1}$ or smaller, so that the distance of C to the edge of the plane can be smaller than $H_i^{-1}/2$. For $l_i = H_i^{-1}$ one can show that less than half of the strings miss \mathcal{V} and give no perturbations to it, so that we can estimate this effect by replacing ν by $\nu_{\text{eff}} = 0.5\nu$. If the strings are moving slowly, so that l_i is even smaller, and not at about the speed of light, our model is not really applicable because the formula for the imparted initial velocity perturbations would change.

Actually a string can affect \mathcal{V} if $c \leq R_i + H_i^{-1}$. But for c larger than H_i^{-1} we encounter the problem mentioned in the previous paragraph, so that we overestimate the perturbations by using $r_{max}^i = H_i^{-1} + R_i$. Therefore we calculate the cumulants for both $r_{max}^i = H_i^{-1}$ and $r_{max}^i = H_i^{-1} + R_i$ and take their average, and the model is more accurate for smaller R . But since at scales below about $5h^{-1}$ Mpc nonlinear effects become important, and we are only considering linear perturbations, we must also keep above that scale.

4 Probability Distribution and Comparison with Observations

First we want to look at the shape of the probability distribution for v_x . The nongaussianity is largest on the smallest scales since smoothing makes things more gaussian,

and larger regions are affected by more strings. Therefore we are going to compare $p(v_x)$ from strings with the results from IRAS at the smallest scale of $R = 6h^{-1}$ Mpc considered in Kofman et al. (1994). Also, the derivation of $M_X(t)$ is valid for scales of $R \leq l_{eq}$, where $l_{eq} = 13h^{-2}$ Mpc is the comoving size of the Hubble radius at the time of equal matter and radiation. The values $\Omega = 1$, $h = 1/2$ and $z_{eq} = 2.3 \cdot 10^4 \Omega h^2$ are used.

Using a symbolic manipulation program (O'Dell 1991), the cumulants are obtained from $M_X(t)$ according to eq.(5), giving the following values for the λ_j (eq.(6)) needed in the expansion of the probability distribution (eq.(8)) for $R = 6h^{-1}$ Mpc :

$$\begin{aligned}\lambda_4 &= 0.34/\nu \\ \lambda_6 &= 0.20/\nu^2 \\ \lambda_8 &= 0.15/\nu^3\end{aligned}\tag{19}$$

These values, as well as the standard deviations quoted below, are the averages of the two cases $r_{max}^i = H_i^{-1}$ and $r_{max}^i = H_i^{-1} + R_i$. For two values of ν , $p(v_x/\sigma)$ is plotted in Figure 1, up to the term involving $H_8(\delta)$ in the expansion (eq.(8)). For the higher value of $\nu = 10$ strings per Hubble volume, the distribution is practically indistinguishable from a gaussian one. For $\nu = 1$ there is a slight deviation. We want to see if this deviation is significant by performing a χ^2 -test with the data given in Kofman et al. (1994), which has been grouped into bins of size $v_x/\sigma = 0.25$. The data points fall practically on a gaussian curve, so instead of taking the exact values from the data we calculate the absolute frequencies m_j in the j bins expected if the underlying distribution were gaussian. The IRAS 1.9Jy survey maps out a sphere of radius $80h^{-1}$ Mpc, so that there are $(80/6)^3$ independent smoothing regions of radius $6h^{-1}$ Mpc. Let n_j be the corresponding absolute frequencies expected from the stringy distribution. Using 20 inner bins, we find $\chi^2 = 4.63$ for $\nu = 1$, where

$$\chi^2 = \sum_{j=1}^{20} \frac{(m_j - n_j)^2}{n_j}\tag{20}$$

, which is much smaller than the 95% confidence upper limit of 38.58 for 19 degrees of freedom, so that the data is in agreement with the probability distribution from strings. If we replace ν by $\nu_{\text{eff}} = \nu/2$ to take into account that the side of the plane swept out by the string in the direction of its motion is only half the diameter of the Hubble volume, then $\chi^2 = 22.5$ for $\nu = 1$.

Next we want to compare the magnitudes of velocities. The standard deviation of the single velocity components is calculated to be

$$\sigma = 1.04\nu^{1/2}\tilde{u} \quad \text{for } R = 6h^{-1}\text{Mpc} \quad (21)$$

$$\sigma = 0.99\nu^{1/2}\tilde{u} \quad \text{for } R = 12h^{-1}\text{Mpc} \quad (22)$$

where $\tilde{u} = 0.4z_{eq}^{1/2}u$, and u is defined in eq.(1). We compare these values with results from Peacock and Dodds (1994), who used the power spectra of various observations to calculate the 3 dimensional rms velocities v_{rms} of regions of radius R . For a gaussian random variable with three independent gaussian variables as components, the standard deviation of a single component is given by $\sigma = v_{rms}/\sqrt{3}$. Using this relation, the values given in Peacock and Dodds (1994) are

$$\sigma = (381 \pm 156) \text{ km/s} \quad \text{for } R = 6h^{-1}\text{Mpc} \quad (23)$$

$$\sigma = (337 \pm 138) \text{ km/s} \quad \text{for } R = 12h^{-1}\text{Mpc} \quad (24)$$

where we have taken the fractional error of $1/\sqrt{6}$ quoted for the actual measurement of v_{rms} at a scale of $5h^{-1}$ Mpc. Comparison of σ in the string model with the values obtained from observations at these two scales, gives as an average value for αf

$$\overline{\alpha f} = 3.2 \pm 0.9 \quad (25)$$

where α is the combination of parameters

$$\alpha = \sqrt{\nu} \frac{G\mu}{10^{-6}} \frac{\gamma_s v_s}{c} \quad (26)$$

α can be constrained from the rms value of the temperature fluctuations in the cosmic microwave background measured by COBE to be (Perivolaropoulos 1993a)

$$\alpha = 1.0 \pm 0.2 \quad (27)$$

Using this value of α , we find

$$\bar{f} = 3.2 \pm 1.1 \quad (28)$$

This indicates that there must be some small scale structure on the strings (see eq.(1)) in order to obtain the right magnitude of peculiar velocities and consistency with CMB observations. This is also in agreement with recent simulations (Bennett & Bouchet 1988, Albrecht & Turok 1989, Allen & Shellard 1990), which show the presence of small-scale structure on cosmic strings. Our analysis has been done in the string wake model, whereas strings with lots of small scale structure accrete matter rather in the form of filaments. Therefore the precise value of f is not to be taken too seriously.

Numerical simulations of peculiar velocities from long strings without small scale structure have been performed in a similar framework (Hara, Mähönen & Miyoshi 1993), where ν' strings are assumed to move across the horizon at every e-fold expansion of the universe. The authors found that

$$\frac{G\mu}{10^{-6}} \frac{\gamma_s v_s}{c} \sqrt{\nu'} = (4 \pm 1) \quad (29)$$

yields good agreement with observations. The number of strings at every two-fold expansion used in our analysis is related to ν' by $\nu = \nu' \ln 2$. Therefore $\alpha f = 3.3 \pm 0.8$ from these simulations, which agrees quite well with our result of $\overline{\alpha f} = 3.2 \pm 0.9$.

5 Discussion

We have shown that the probability distribution of a single component of peculiar velocities in the cosmic string wake model is very close to a normal distribution

on scales of several h^{-1} Mpc, as suggested by a general argument for seed models (Scherrer 1992), and in agreement with observations. A comparison of the measured magnitude of peculiar velocities with that expected from strings, using the normalization of string parameters from the COBE quadrupole, suggested that strings have some small-scale structure. Nongaussian features are more apparent in the velocity differences than in the velocities themselves (Catelan & Scherrer 1994), and it would be interesting to calculate the probability distribution of these velocity differences within the cosmic string model of structure formation.

6 Acknowledgements

I would like to thank Robert Brandenberger and Leandros Perivolaropoulos for suggestions and helpful discussions. This work was supported in part by the US Department of Energy under Grant DE-FG0291ER40688.

References

- Albrecht A., Turok N., 1989, Phys. Rev. D 40 , 973.
- Allen B., Shellard E.P.S, 1990, Phys. Rev. Lett. 64, 119.
- Bennett D., Bouchet F., 1988, Phys. Rev. Lett. 60, 257.
- Bennett D., Bouchet F., Stebbins A., 1992, ApJ (Lett.) 399, L5.
- Brandenberger R., 1991, Phys. Scripta T36, 114.
- Brandenberger R., Perivolaropoulos L., Stebbins A., 1990, Int. J. Mod. Phys. A5, 1633.
- Brandenberger R. et al., 1987, Phys. Rev. D36, 335.
- Catelan P., Scherrer R., 1994, "Velocity Differences as a Probe of Non-Gaussian Density Fields" (preprint SISSA Ref. 37/94/A).
- Erdelyi A., 1956, "Asymptotic Expansions", (Dover).
- Hara T., Mähönen P., Miyoshi S., 1993, ApJ 415, 445.
- Hara T., Miyoshi S., 1990, Prog. Theor. Phys. 81, 1187.
- Kofman L. et al., 1994, ApJ 420, 44.
- Moessner R., Perivolaropoulos L., Brandenberger R., 1994, ApJ 425, 365.
- Nusser A. et al., 1991, ApJ 379, 6.
- O'Dell J., 1991, "ALJABR", Fort Pond Research.
- Peacock J., Dodds S., 1994, MNRAS 267, 1020 .
- Perivolaropoulos L., 1993a, Phys. Lett. B 298, 305.
- Perivolaropoulos L., 1993b, Phys. Rev. D 48, 1530.
- Perivolaropoulos L., Brandenberger R., Stebbins A., 1990, Phys. Rev. D 41, 1764.
- Perivolaropoulos L., Vachaspati T., 1994, ApJ 423, L77.
- Sato H., 1986, Prog. Theor. Phys. 75, 1342.
- Scherrer R., 1992, ApJ 390, 330.
- Scherrer R., Bertschinger E., 1991, ApJ 381, 349.
- Silk J., Vilenkin A., 1984, Phys. Rev. Lett. 53, 1700.
- Stebbins A., 1986, ApJ (Lett.) 303, L21.

- Stebbins A. et al., 1987, ApJ 322, 1.
- Strauss M. et al., 1990, ApJ 361, 49.
- Strauss M. et al., 1992, ApJ 385, 421.
- Stuart A., Ord J., 1987, "Kendall's Advanced Theory of Statistics", Vol.1 (London: Charles Griffin).
- Traschen J., Turok N., Brandenberger R., 1986, Phys. Rev. D 34, 919.
- Turok N., 1991, Phys. Scripta T36, 135.
- Turok N., Brandenberger R., 1986, Phys. Rev. D 33, 2175.
- Vachaspati T., 1986, Phys. Rev. Lett. 57, 1655.
- Vachaspati T., 1992, Phys. Lett. B, 282, 305.
- Vachaspati T., Vilenkin A., 1991, Phys. Rev. Lett. 67, 1057.
- Veeraraghavan S., Stebbins A., 1992, ApJ Lett. 395, L55.
- Vilenkin A., 1985, Phys. Rep. 121, 263.
- Vollick D., 1992, Phys. Rev. D 45, 1884.
- Yahil A. et al., 1991, ApJ 372, 380.

Figure Captions

Figure 1: Stringy probability distribution $p(\delta = v_x/\sigma)$ of a single velocity component smoothed over regions of radius $R = 6h^{-1}$ Mpc for $\nu = 1$ (solid line) and $\nu = 10$ strings per Hubble volume (dotted line), compared with a normal distribution (dashed line).

solid line: $\nu=1$, dotted line: $\nu=10$, dashed line: Gaussian

